

# $\Lambda_\nu^\mu$ geometries from the point of view of different observers

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$\Lambda_\nu^\mu$ -geometry is a geometry with a variable cosmological term described by a second-rank symmetric tensor  $\Lambda_\nu^\mu$  whose asymptotics are Einstein cosmological term  $\Lambda\delta_\nu^\mu$  at the origin and  $\lambda\delta_\nu^\mu$  at infinity (with  $\lambda < \Lambda$ ). It corresponds to extension of the algebraic structure of the Einstein cosmological term  $\Lambda\delta_\nu^\mu$  in such a way that a scalar  $\Lambda$  describing vacuum energy density as  $\rho_{vac} = 8\pi G\Lambda$  (with  $\rho_{vac} = \text{const}$  by virtue of the Bianchi identities), becomes explicit related to the appropriate component,  $\Lambda_0^0$ , of an appropriate stress-energy tensor,  $T_\nu^\mu = 8\pi G\Lambda_\nu^\mu$  whose vacuum properties follow from its symmetry,  $T_0^0 = T_1^1$ , and whose variability follows from the contracted Bianchi identities. In the spherically symmetric case existence of such geometries in frame of GR follows from imposing on Einstein equations requirements of finiteness of the ADM mass  $m$ , and of regularity of density and pressures. Dependently on parameters  $m$  and  $q = \sqrt{\Lambda/\lambda}$ ,  $\Lambda_\nu^\mu$  geometry describes five types of configurations. We summarize here the results which tell us how these configurations look from the point of view of different observers: a static observer, a Lemaitre co-moving observer, and a Kantowski-Sachs observer.

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## I. INTRODUCTION

The motivation for introducing a geometry able to model Lambda-variability is evident: With great probability we live in  $\lambda$ -dominated universe with  $\lambda$  contributing via  $\rho_{vac} = 8\pi G\lambda$  to the total energy density with  $\rho_{vac} \simeq 0.73\rho_{total}$  [1]. On the other hand, the big value of Lambda is responsible for an earliest inflationary stage solving various puzzles of the standard cosmology [2].

The Einstein cosmological term corresponds to a maximally symmetric vacuum stress-energy tensor [3,4]

$$\Lambda\delta_\nu^\mu = 8\pi G\rho_{vac}\delta_\nu^\mu \quad (1)$$

A variable cosmological term is introduced as an extension of  $\Lambda\delta_\nu^\mu$  to a second-rank symmetric tensor  $\Lambda_\nu^\mu$  connecting smoothly two asymptotic maximally symmetric states (1) with different values of cosmological constant [5] (for review [6,7])

$$\Lambda\delta_\nu^\mu \leftarrow \Lambda_\nu^\mu \rightarrow \lambda\delta_\nu^\mu \quad (2a)$$

corresponding to certain vacuum tensor

$$\rho_{vac}^{in}\delta_\nu^\mu \leftarrow T_\nu^\mu \rightarrow \rho_{vac}^{out}\delta_\nu^\mu \quad (2b)$$

Kind of justification for identification of eq.(2) comes from nice discussion in Ref. [8] about where to put the cosmological term in the Einstein equation

$$G_\nu^\mu + \Lambda\delta_\nu^\mu = 0 \quad (3)$$

According to [8] putting it on the right-hand side of the Einstein equation as  $T_\nu^\mu = 8\pi G\rho_{vac}\delta_\nu^\mu$  and treating as a dynamical part of a matter content, originates from dialectic materialism of the Soviet physics school where

it came from [3,9], although the first calculation relating  $\Lambda\delta_\nu^\mu$  to a QFT vacuum was done by De Witt [4]. On contrary keeping  $\Lambda\delta_\nu^\mu$  on the left-hand side and treating it as geometrical entity with  $\Lambda$  as a fundamental constant of nature, is preferred by idealistic approach.

The first approach - shifting  $\Lambda\delta_\nu^\mu$  to the right side as some  $T_\nu^\mu$ , allows energy exchange of de Sitter vacuum (1) with another matter. Even a simplest nonsingular FRW cosmological model with the initial de Sitter state and conversion of a vacuum energy into radiation, gives (generically, for any source of  $\Lambda$ ) needed entropy and accelerated exponential expansion with e-folding number sufficient to explain homogeneity, isotropy, present size and density of the Universe [10] (for review [2]).

Such an approach is most popular till now for making  $\Lambda$ -variability due to exchange with other matter in various models [11,8], and for solving problem of cosmological constant in field theories by cancelling pre-existing vacuum energy (cosmological constant) with developed by an appropriate field big expectation value of a vacuum density [12] (for review [13]).

The aim of the reported research was to reveal how one could make cosmological term variable in itself.

The approach in introducing (2) was in a sense opposite to that classified as materialistic. We first found in the right-hand side of the Einstein equation

$$G_\nu^\mu = -8\pi GT_\nu^\mu \quad (4)$$

the class of source terms with the proper symmetry [14]

$$T_t^t = T_r^r; \quad T_\theta^\theta = T_\phi^\phi \quad (5)$$

and asymptotic behavior [15]

$$\Lambda\delta_\nu^\mu \leftarrow 8\pi GT_\nu^\mu \rightarrow \lambda\delta_\nu^\mu \quad (6)$$

shift it then to the left-hand side of equation (4) as [5]

$$\Lambda_\nu^\mu = 8\pi G T_\nu^\mu \quad (7)$$

treating as evolving and clustering geometrical entity.

The cosmological term (7) is invariant under radial boosts and is identified as corresponding to a spherically symmetric anisotropic vacuum [14], with the algebraic structure [(II)(II)] in the Petrov classification.

The existence of the class of regular spherical metrics generated by stress-energy tensors of structure (5) follows from requirements of regularity of density  $= \rho$  and finiteness of the ADM mass  $m$ , and imposing dominant energy condition on a stress-energy tensor [16,7], either weak energy condition and regularity of pressures [17]. These two possibilities correspond to geometries with variable curvature scalar  $R$ , positive in the first case [17].

At present there is known a lot of matter sources contributing to GR equations with a stress-energy tensor of structure (1). All of them give the same geometry - de Sitter geometry governed by  $\Lambda\delta_\nu^\mu$  - whose generic properties are used then in relevant physical models [2].

In similar way we study geometries generated by (7) whose mathematical properties are generic. The advantage is using of possibilities given by mathematical models followed from GR equations, to obtain a Lambda-dominated stage at which a matter comes into play.

In this talk we show how the vacuum configurations represented by  $\Lambda_\nu^\mu$  geometry, look from the point of view of different observers: a static observer, a co-moving Lemaitre observer and a Kantowski-Sachs observer.

## II. $\Lambda_\nu^\mu$ GEOMETRY

The Einstein cosmological term (1) is identified as a vacuum stress-energy tensor due to its maximally symmetric form. It is invariant under any coordinate transformation which makes impossible to distinguish a preferred co-moving reference frame [3]. As a result an observer moving through a medium with a stress-energy tensor (1) cannot in principle measure his velocity with respect to it, which allows one to classify it as a vacuum in accordance with the relativity principle [3].

Introducing in similar way a vacuum with variable density, one cannot keep the full invariance which leads, by Bianchi identities, to  $\rho_{vac} = \text{const.}$  The invariance can be kept for an observer moving along a certain direction in space distinguished by symmetry of a source term (5). In  $\Lambda_\nu^\mu$  extension of  $\Lambda\delta_\nu^\mu$  a constant scalar  $\Lambda$  associated by (1) with a vacuum density, constant by virtue of Bianchi identities, becomes a tensor component  $\Lambda_t^t = 8\pi G T_t^t$  associated explicitly with a density component of a perfect fluid stress-energy tensor, whose vacuum properties follow from its symmetry and whose variability follows just from the Bianchi identities [5].

For the stress-energy tensor of the algebraic structure (5) the generalized Birkhoff theorem [18] guarantees the existence of a coordinate frame where the metric has the static form

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2 d\Omega^2 \quad (8)$$

where  $d\Omega^2$  is the line element on the unit 2-sphere. Here we concentrate on the spherically symmetric case although generalization is straightforward to the cases of 2-dimensional planes and Lobachevsky planes [19].

For the case of  $\Lambda_\nu^\mu$  with asymptotic behavior (2), the metric function is given by [15]

$$g(r) = 1 - \frac{R_g(r)}{r} - \frac{\lambda}{3}r^2 \quad (9)$$

with

$$R_g(r) = 2GM(r); \quad M(r) = 4\pi \int_0^r \rho(r)r^2 dr \quad (10)$$

The asymptotic value of lambda at infinity is included into stress-energy tensor in such a way that

$$T_t^t = \rho(r) + (8\pi G)^{-1}\lambda \quad (11)$$

so that the ADM mass  $m$  is defined in the standard way

$$m = 4\pi \int_0^\infty \rho(r)r^2 dr \quad (12)$$

From the conserved Bianchi identities it follows the conservation equation  $\Lambda_{\nu;\mu}^\mu = 0$  which gives the equation of state connecting components of a cosmological term  $\Lambda_t^t = 8\pi G\rho(r)$ ,  $\Lambda_r^r = -8\pi Gp_r$ ,  $\Lambda_\theta^\theta = \Lambda_\phi^\phi = -8\pi Gp_\perp$  [5]

$$p_r = -\rho; \quad p_\perp = p_r + \frac{r}{2} \frac{dp_r}{dr} \quad (13)$$

In the case of two scales for vacuum density, geometry has not more than three horizons [19], and describes five types of configurations as shown in Fig.1 [15].

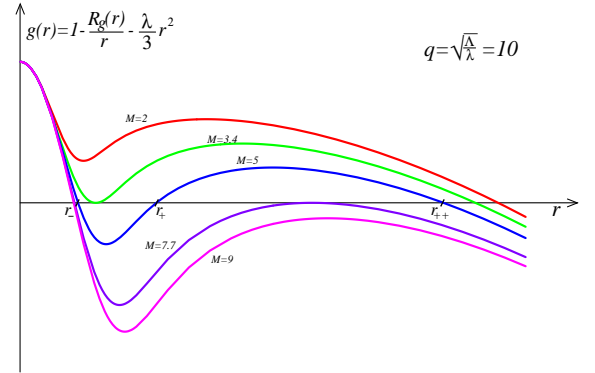


FIG. 1. The metric  $g(r)$  for  $\Lambda_\nu^\mu$  configurations. The mass  $m$  is normalized to  $(3/G^2\Lambda)^{1/2}$ . The parameter  $q = \sqrt{\Lambda/\lambda}$ .

The case of three horizons, an internal horizon  $r_-$ , a black (white) hole horizon  $r_+$ , and a cosmological horizon  $r_{++}$ , corresponds to the nonsingular modification of Kottler-Treftz solution [20] referred to in the literature as Schwarzschild-de Sitter geometry [21]. In the regular version it exists only within a certain range of mass parameter,  $m_{min} \leq m \leq m_{max}$ , with limits depending on the value of the parameter  $q = \sqrt{\Lambda/\lambda}$  [15].

In  $\Lambda^\mu$  geometries singularities  $r = 0$  are replaced with a regular R-regions asymptotically de Sitter with the value of  $\Lambda$  as  $r \rightarrow 0$  corresponding to a scale of symmetry restoration [22,16]. In these regions  $g(r) > 0$ . The regions where  $g(r) < 0$  are called T-regions.

R- and T- regions are specified by the invariant quantity  $\Delta = g^{\mu\nu} r_{,\mu} r_{,\nu}$  (see, e.g., [23]). In R-regions  $\Delta < 0$ , the surfaces  $r=\text{const}$  are time-like, static observers can exist, move and send signals in both directions. In T-regions  $\Delta > 0$ , the surfaces  $r=\text{const}$  are space-like, no static observer can exist in principle, both signals from this surface propagate in the same direction. T-regions are regions of one-way traffic. In Fig. 1 they are located between horizons  $r_-$ ,  $r_+$  and between  $r_{++}$  and infinity. At horizons  $\Delta = 0$ . For the metric in the Kruskal form  $\Delta = (1/2)g(r)^{-1}r_{,u}r_{,v}$ , and the conditions  $r_{,u} > 0$  and  $r_{,u} < 0$  are invariant:  $r_{,u} < 0$  for contracting  $T_-$  region;  $r_{,u} > 0$  for an expanding  $T_+$  region.

### Static observers

Static observers exist only in R-regions. In all types of configurations horizons exist, at least one related to replacing a singularity with a de Sitter core. In most general case of three horizons (see Fig.1), a static observer between  $r_+$  and  $r_{++}$  observes a vacuum nonsingular cosmological black (white) hole, and his horizons are black hole horizon  $r_+$  and cosmological horizon  $r_{++}$  in his future (past). A static observer between  $r = 0$  and  $r = r_-$  sees internal horizon  $r_-$  as his cosmological (future or past) horizon. It is seen from Fig.2 which presents the global structure of  $\Lambda_{\mu\nu}$  geometry with three horizons [15].

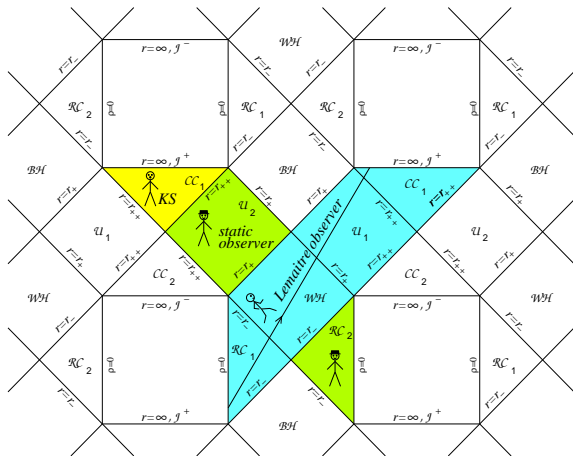


FIG. 2. The global structure of space-time with 3 horizons.

Static observers see Hawking radiation from black hole horizons, and Gibbons-Hawking radiation [21] from cosmological horizons, with the temperature [24]

$$kT = \frac{\hbar c}{4\pi} \left[ \frac{R_g(r_h)}{r_h^2} - \frac{R'_g(r_h)}{r_h} \right]; \quad r_h = r_-, r_+, r_{++}$$

### Kantowski-Sachs observers

In T-regions the coordinates  $r$  and  $t$  interchange their roles,  $r$  becomes time-like and  $t$  space-like. The metric (8) can be re-written, introducing time coordinate  $u$ , space-like coordinate  $x$  and the metric function is  $a^2(u) = -g(u)$ , in the form [19]

$$ds^2 = \frac{1}{a^2(u)} du^2 - a^2(u) dx^2 - u^2 d\Omega^2 \quad (14)$$

It describes an anisotropic cosmological model with two time-dependent scale factors  $a(u)$  and  $b(u) = u$ , and the lapse function  $1/a^2(u)$ . A spatial section has topology of a 3-dimensional cylinder with different time-dependent scale factors in the radial and longitudinal directions [19].

Models described by metric (14) belong to Kantowski-Sachs type homogeneous anisotropic cosmological models with two time-dependent scale factors [25]. Kantowski-Sachs models represent a special class of T-models [26] which are in general inhomogeneous.

Essentially new feature of these models in our case is the existence of regular R-region near  $r = 0$ , so metrics (14) are identified as Kantowski-Sachs models with the regular R-regions [19].\*

These models can describe both regular cosmologies in  $T_+$ -regions and regular collapse in  $T_-$ -regions.

For Kantowski-Sachs observers the cosmological evolution starts from horizons  $u = r_h$  which are highly anisotropic purely coordinate singularities (the 4-geometry is perfectly globally regular), where coordinate surfaces, spheres with the same finite scale factor  $u(r_h)$  stick to one another. As a result cosmological evolution for Kantowski-Sachs observers starts with a **null bang from a horizon**, a null surface ( $u = r_h$ ) with vanishing volume of spatial section squeezed along  $x(a(r) = 0)$ ; this happens at finite cosmological time  $\tau(u) = \int du/a(u)$  for the case of a simple horizon, and in the infinitely remote past for higher-order horizons [19].

In our case Kantowski-Sachs models have regular R-regions, and Kantowski-Sachs observers can receive information from their remote past (even in the case of high-order horizons) brought by particles and photons crossing a horizon in their finite proper time [19].

\*In the case of pseudospherical symmetry 2-space is Lobachevsky plane models are identified as hyperbolic Kantowski-Sachs models [19]. In the case of the planar symmetry of 2-space, models belong to Bianchi type I [19].

### Lemaitre observers

As usual in space-times with horizons, a static observer can see only small part of the manifold, R-region in which he is actually resided. The same concerns Kantowski-Sachs observer who exists only in T-regions. Both have problems with horizons which are singular surfaces for them. Removing coordinate singularities in the way similar to applied by Lemaitre for Schwarzschild geometry, one introduces coordinates of an observer to whom more extended parts of a manifold are available.

Connecting coordinates with the particles moving on radial geodesics marked by the constant of motion  $E^2$ , we make transformation  $r, t \rightarrow R, \tau$  where  $R, \tau$  are coordinates of co-moving observers in which the metric (8) takes the Lemaitre form [19]

$$ds^2 = d\tau^2 - \frac{(E^2 - g(r(R, \tau)))}{E^2} dR^2 - r^2(R, \tau) d\Omega^2 \quad (15)$$

The models described by (15), belong to the Lemaitre class of cosmological models with anisotropic perfect fluid, since the principal pressures are essentially different for  $\Lambda_\nu^\mu$  geometry satisfying the equation of state (13). In coordinates connected with in-falling particles, the metric (15) describes a regular gravitational collapse.

For all  $\Lambda_\nu^\mu$  configurations, Lemaitre class cosmologies describe evolution starting with a **nonsingular non-simultaneous de Sitter bang** [27,19].

It can be easily seen for the case of de Sitter-Schwarzschild geometry [24] which is the particular case of  $\Lambda_\nu^\mu$  geometry with  $\lambda = 0$ . The global structure of de Sitter-Schwarzschild space-time is shown in Fig.3 where it is compared with the case of Schwarzschild geometry.

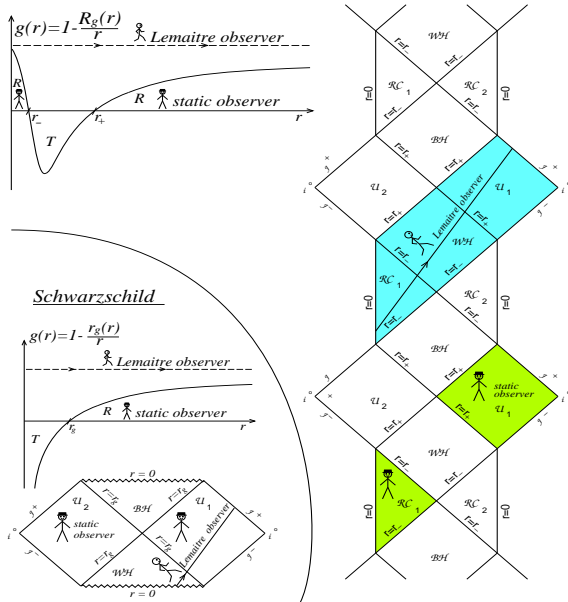


FIG. 3. De Sitter-Schwarzschild space-time.

Lemaitre coordinates  $R, \tau$  map the segment  $\mathcal{RC}$  (regular core),  $\mathcal{WH}$  (white hole),  $\mathcal{U}$  (universe) available to Lemaitre observers. A regular core  $\mathcal{RC}$  models an initial state for an expanding universe in all  $\Lambda_\nu^\mu$  configurations. Evolution starts from the surface  $r(R, \tau) = 0$  which is the bang surface. In Schwarzschild case this is the surface of big bang singularity [28]. For example, in the case of the Tolman-Bondi dust model, it is described by  $r(R, \tau) = (9M(R)/2)^{1/3}(\tau - \tau_0(R))^{2/3}$  with the bang-time function  $\tau_0(R)$  [29].

In the case of Schwarzschild white hole the bang surface  $c\tau + R = 0$  (see Fig.4) is space-like surface (see Fig.3). In de Sitter-Schwarzschild geometry the bang surface ( $c\tau + R = -\infty$  in Fig.4) is the time-like regular surface ( $r = 0$  in Fig.3).

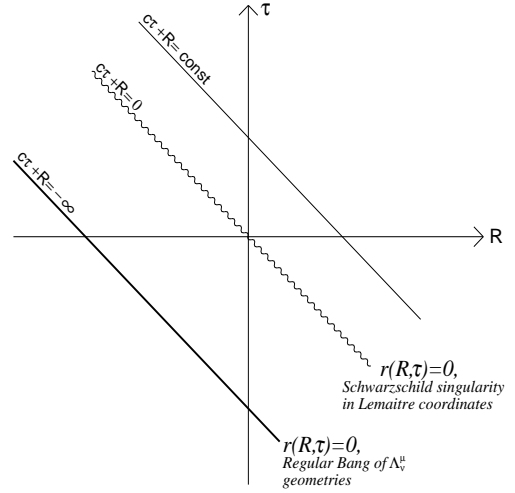


FIG. 4. The surfaces  $r=\text{const}$  in Lemaitre coordinates.

Since a bang surface is timelike, different points start at different moments of the synchronous time  $\tau$ .

Near the bang surface the metric (15) takes the FRW form

$$ds^2 = d\tau^2 - a^2(\tau)(d\chi^2 + \sin^2\chi d\Omega^2) \quad (16)$$

with the de Sitter scale factor  $a(\tau) \sim \exp(H\tau)$  for a spatially flat model,  $a(\tau) \sim \cosh(H\tau)$  for a closed model, and  $a(\tau) \sim \sinh(H\tau)$  for an open model;  $H$  is the Hubble parameter  $H = \sqrt{\Lambda/3}$  [27].

An inflationary stage is followed by a Kasner-type anisotropic stage, with contraction in the radial direction and expansion in the tangential direction, at which most of a universe mass is produced [27].

The metric (15) at the Kasner-type stage takes the form [27,19]

$$ds^2 = d\tau^2 - (\tau + R)^{-2/3} F(R) dR^2 - B(\tau + R)^{4/3} d\Omega_1^2 \quad (17)$$

where  $F(R)$  is a smooth regular function and  $B$  is a constant related to the model parameters. At this stage acceleration is changes drastically (see Fig.5 [27]).

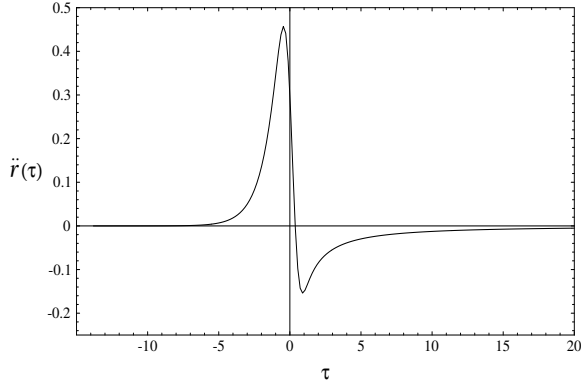


FIG. 5. Acceleration of the scale factor  $r(\tau)$  [27].

At late times all  $\Lambda_\nu^\mu$  dominated models become isotropic and approach de Sitter asymptotic described by (16) with the Hubble parameter  $H = \sqrt{\Lambda/3}$  [19].

In the case  $E^2 = 1$  models asymptotically approach flat FRW models in both past  $\Lambda$  and future  $\lambda$  limits; a de Sitter bang occurs in infinitely remote past,  $R + c\tau = -\infty$ . In the case  $E^2 > 1$ , when the initial velocity on the geodesics of the reference frame is nonzero, models asymptotically approach open FRW models; a bang occurs at finite  $R + \tau$  [19]. In case  $E^2 < 1$  (asymptotically approaching closed FRW model) a bang starts from a finite value  $r_{in}$  given by  $E^2 - g(r_{in}) = 0$  at finite  $R + \tau$ .

### III. POSSIBILITIES GIVEN BY $\Lambda_{\mu\nu}$ GEOMETRY

#### One horizon configurations

There are two types of one-horizon configurations shown in Fig.6. In both cases the global structure of space-time is the same as for de Sitter geometry. Essential difference is that cosmological density  $\Lambda_t^t$  in  $\Lambda_{\mu\nu}$  geometry evolves smoothly from  $\Lambda$  to  $\lambda$ . This is evolution in  $r$  for a static observers in R-region, in proper time  $\tau$  for Lemaitre and Kantowski-Sachs observers.

For Lemaitre observers evolution starts from non-simultaneous de Sitter bang  $r(R, \tau) = 0$ . For Kantowski-Sachs observers evolution starts with a null bang from a single horizon, in their finite proper time.

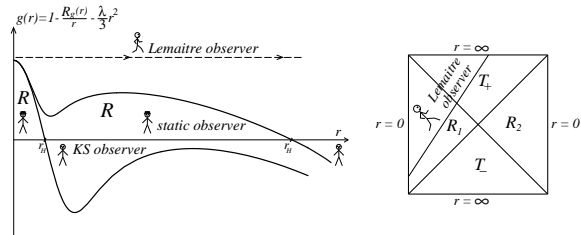


FIG. 6.  $\Lambda_\nu^\mu$  geometry with one horizon.

In the first case, the upper curve in Fig.6, the horizon is related to the small asymptotic value,  $\lambda$ ,  $r_- \approx \sqrt{\lambda/3}$ . Observers in R-region see Hawking radiation with the Gibbons-Hawking temperature  $T \sim \hbar c \sqrt{\lambda/3}$ . The essential dynamical changes occur in the R-region, experienced by static observers, or by comoving Lemaitre observers before crossing horizon.

In second case, the lower curve in Fig.6, the horizon is approximately related to big asymptotic value,  $\Lambda$ ,  $r_{++} \approx \sqrt{\Lambda/3}$ . Observers see Hawking radiation with the Gibbons-Hawking temperature  $T \sim \hbar c \sqrt{\Lambda/3}$ . One can say that they are in much hotter environment. The essential dynamical changes occur in T-region as dynamical possibilities for Kantowski-Sachs observers.

#### Configuration with three horizons

This configuration is shown in Fig.7 [15].

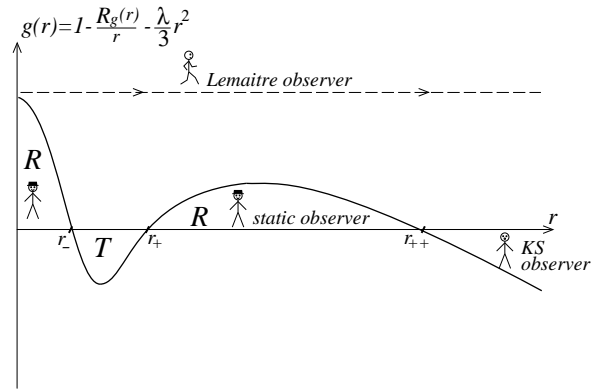


FIG. 7.  $\Lambda_\nu^\mu$  geometry with three horizons.

The global structure of spacetime is shown in Fig.2.

An observer in R-region between  $r_+$  and  $r_{++}$  observes Hawking radiation from both black hole and cosmological horizons, whose temperature in certain range of parameters can be estimated as  $kT \sim \hbar c^3/8\pi Gm$  for black hole horizon and by  $kT \sim \hbar c \sqrt{\lambda/3}$  for a cosmological horizon. An observer in R-region between  $r = 0$  and  $r = r_-$  observes Gibbons-Hawking radiation with temperature estimated roughly by  $kT \sim \hbar c \sqrt{\Lambda/3}$ .

The segments  $\mathcal{RC}, \mathcal{WH}, \mathcal{U}, \mathcal{CC}$ , are available to Lemaitre observers. They can cross horizons without big problems (tidal forces are big only when  $m$  is small). Coordinate frame related to radially infalling observers describes nonsingular contraction, while frames related to outgoing geodesics describe cosmological models of Lemaitre class with a non-simultaneous de Sitter bang.

T-regions of a black (white) hole between  $r_-$  and  $r_{++}$  and cosmological regions between  $r_{++}$  and infinity are regions of one-way traffic. The cosmological regions are the place of Kantowski-Sachs observers. From their point of view evolution starts from a horizon  $r_{++}$  with a null bang in their remote past but in finite proper time.

Even in such a complicated space-time KS observers can receive information about pre-bang history brought by particles and photons from the first R-region, first  $T_+$ -region which in this case is a white hole, and next R-region (universe of a static observer).

#### Double horizon $r_+ = r_{++}$

This configuration, the first extreme state of a nonsingular cosmological black hole (nonsingular modification of the Nariai geometry [30]), is shown in Fig.8.

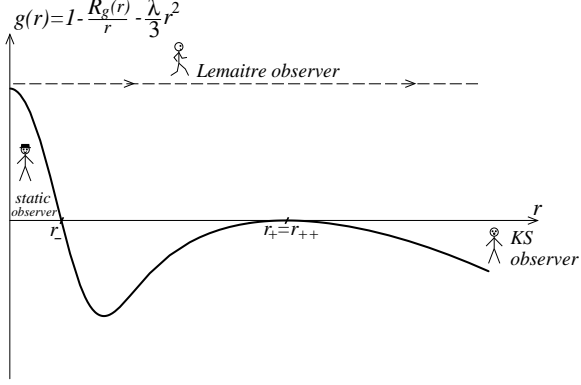


FIG. 8.  $\Lambda_\nu^\mu$  geometry with a double horizon  $r_+ = r_{++}$ .

Global structure of space-time is shown in Fig.9 [19].

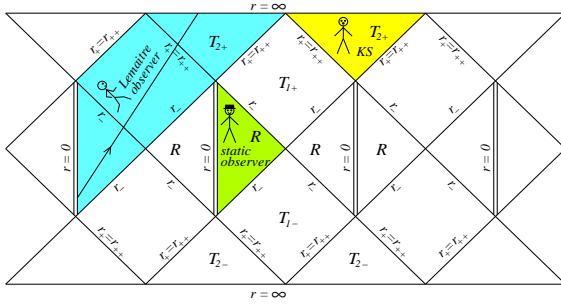


FIG. 9. The global structure of  $\Lambda_\nu^\mu$  space-time with a double horizon  $r_+ = r_{++}$ .

For observers in R-region between  $r = 0$  and  $r = r_-$ , horizon is dominated by the big value  $\Lambda$ . The Gibbons-Hawking temperature  $T \sim \hbar c \sqrt{\Lambda/3}$ .

Coordinate frame of Lemaitre observers map the segment which contains R-region and two T-regions.

The surface  $r_+ = r_{++}$  is the null bang for Kantowski-Sachs observers in  $T_+$ -regions between  $r_+ = r_{++}$  and infinity, the bang occurs in their infinitely remote past. Nevertheless, pre-bang information arrives to Kantowski-Sachs observers, since for any other geodesics with  $E^2 > 1$  in Kantowski-Sachs part of a manifold, a time for arriving at any finite value of  $r$  is finite [19].

For in-falling KS observers in  $T_-$  regions, an apparent coordinate singularity is in their infinitely remote future.

#### Double horizon $r_- = r_+$

This configuration is shown in Fig.10, global structure of space-time in Fig.11. For a static observer in R-region between  $r_- = r_+$  this is nonsingular ( $\Lambda$  as  $r \rightarrow 0$ ), cosmological ( $\lambda$  as  $r \rightarrow \infty$ ), extreme (double horizon) black hole.

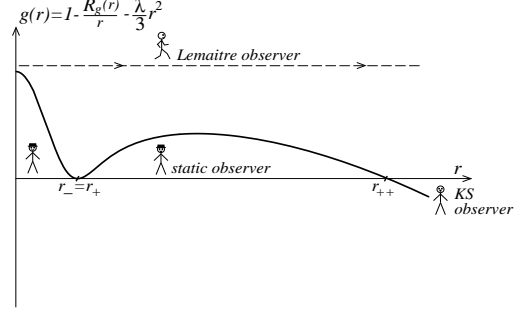


FIG. 10.  $\Lambda_\nu^\mu$  geometry with a double horizon  $r_- = r_+$ .

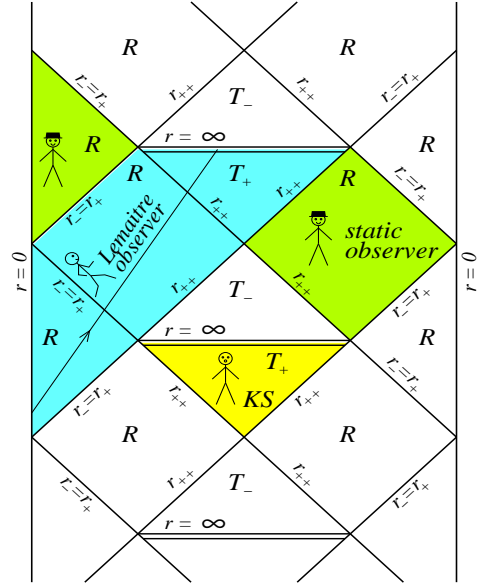


FIG. 11. The global structure for  $\Lambda_\nu^\mu$  space-time with a double horizon  $r_- = r_+$ .

This extreme black hole state has no analogy in singular models, it appears because of replacing a black hole singularity with an asymptotically de Sitter core which produces an additional internal horizon, and a lower bound on a black hole mass comes as a result [24]. When a black hole approaches this limit, Hawking temperature from the BH horizon goes to zero [24]. An observer in R-region between  $r_- = r_+$  and  $r_{++}$  sees only Hawking radiation from his cosmological horizon  $r_{++}$ .

Segment of a manifold available for Lemaitre observers includes two R-regions and one T-region.

For Kantowski-Sachs observers in T-region evolution starts with a null bang in finite time in their past.



#### IV. SUMMARY AND DISCUSSION

Dependently on the number of horizons,  $\Lambda_\nu^\mu$  geometries describe five types of globally regular configurations, seen by a static observer as cosmological vacuum nonsingular black hole with an additional internal horizon separating an additional R-region, its two extreme states, and two types of one horizon configurations with global structure of de Sitter geometry but with vacuum energy  $\Lambda_t^t = 8\pi G\rho_{vac}$  smoothly evolving from  $\Lambda$  to  $\lambda$ .

All  $\Lambda_\nu^\mu$  cosmological models belong to the Lemaitre class of models with anisotropic perfect fluid. Cosmological evolution for Lemaitre co-moving observers starts with a non-singular non-simultaneous de Sitter bang which is followed by a Kasner-type stage of anisotropic expansion. At late times all models approach de Sitter asymptotic with small  $\lambda$ .

Spherically symmetric  $\Lambda_\nu^\mu$  models of Kantowski-Sachs type contain the regular R-regions. For contracting  $T_-$ -regions evolution finishes and for expanding  $T_+$ -regions starts with the null bang from horizons, which is in a finite proper time of Kantowski-Sachs observers in the case of a simple horizon, and in their infinitely remote past in the case of a higher-order horizon.

Kantowski-Sachs observers get information about pre-bang history brought by particles and photons which have crossed the horizon in their finite proper time.

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